Komplexe Algebra

Übersicht

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Definitionen

$Z, X, Y, ...$: Komplexe Zahlen, $Z, X, Y, ... \in \mathbb{C}$

Basis Beziehungen

$\boxed{j^2 = -1}$

$\boxed{e^{j\phi} = \cos(\phi) + j\sin(\phi)}$: Eulersche Formel

$Z = a + jb = re^{j\phi} = r\cos(\phi) + jr\sin(\phi)$

Kartesische Koordinaten in Polarkoordinaten

$\boxed{Z : Z = a + jb \to Z = re^{j\phi}}$

$\boxed{r := |Z| = \sqrt{a^2 + b^2}}$ r : Betrag von Z

$\boxed{\phi := \arctan(\frac{b}{a})}$ ϕ : Phase von Z

Polarkoordinaten in Kartesische Koordinaten

$\boxed{Z : Z = re^{j\phi} \to Z = a + jb}$

$\boxed{a = r\cos(\phi)}$ a : Realanteil von Z

$\boxed{b = r\sin(\phi)}$ b : Imaginaeranteil von Z

Addition

$Z = X + Y = a_x + jb_x + a_y + jb_y = (a_x + a_y) + j(b_x + b_y)$

$\boxed{Z = X + Y = (a_x + a_y) + j(b_x + b_y)}$

Subtraktion

$$Z = X - Y = a_x + jb_x - a_y - jb_y = (a_x - a_y) + j(b_x - b_y)$$

$$\boxed{Z = X - Y = (a_x - a_y) + j(b_x - b_y)}$$

Multiplikation

Kartesische Koordinaten:

$$Z = X \cdot Y = (a_x + jb_x) \cdot (a_y + jb_y) = (a_x a_y - b_x b_y) + j(a_x b_y + a_y b_x)$$

$$\boxed{Z = X \cdot Y = (a_x a_y - b_x b_y) + j(a_x b_y + a_y b_x)}$$

Polarkoordinaten:

$$Z = X \cdot Y = r_x e^{j\phi_x} \cdot r_y e^{j\phi_y} = r_x r_y e^{j(\phi_x + \phi_y)}$$

$$\boxed{Z = X \cdot Y = r_x r_y e^{j(\phi_x + \phi_y)}}$$

Division

Kartesische Koordinaten:

$$Z = X : Y = \frac{X}{Y} = \frac{a_x + jb_x}{a_y + jb_y} = \frac{(a_x + jb_x)(a_y - jb_y)}{(a_y + jb_y)(a_y - jb_y)} = \frac{a_x a_y + b_x b_y + j a_y b_x - j a_x b_y}{a_y^2 + b_y^2}$$

$$\boxed{Z = X : Y = \frac{X}{Y} = \frac{a_x a_y + b_x b_y}{a_y^2 + b_y^2} + j\frac{a_y b_x - a_x b_y}{a_y^2 + b_y^2}}$$

Polarkoordinaten:

$$Z = X : Y = \frac{X}{Y} = \frac{r_x e^{j\phi_x}}{r_y e^{j\phi_y}} = \frac{r_x}{r_y} e^{j(\phi_x - \phi_y)}$$

$$\boxed{Z = X : Y = \frac{X}{Y} = \frac{r_x}{r_y} e^{j(\phi_x - \phi_y)}}$$

Multiplikation mit konjugiert komplexer Zahl

$$\boxed{Z = X \cdot \overline{X} = (a + jb) \cdot (a - jb) = a^2 + b^2}$$